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On the taxonomy of flux vacua

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Abstract: We investigate several predictions about the properties of IIB flux vacua on Calabi-Yau orientifolds, by constructing and characterizing a very large set of vacua in a specific example, an orientifold of the Calabi-Yau hypersurface in $WP^4_{1,1,1,1,4}$. We find support for the prediction of Ashok and Douglas that the density of vacua on moduli space is governed by $\det(-R-\omega)$ where R and ω are curvature and Kähler forms on the moduli space. The conifold point $\psi = 1$ on moduli space therefore serves as an attractor, with a significant fraction of the flux vacua contained in a small neighborhood surrounding $\psi = 1$. We also study the functional dependence of the number of flux vacua on the D3 charge in the fluxes, finding simple power law growth.

Keywords: F-Theory, Superstring Vacua.

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Contents

1. Introduction

Since the classic work of Candelas, Horowitz, Strominger and Witten in the mid 1980s [1], Calabi-Yau compactification of 10d supergravity has been the dominant paradigm for the construction of 4d string models. In the heterotic string theory, such compactifications yield $\mathcal{N}=1$ supersymmetric theories below the KK scale, and can quite plausibly accomodate the standard model. In the type II context, Calabi-Yau compactification yields $\mathcal{N}=2$ supersymmetric models, which have been of considerable interest in studies of string duality and mirror symmetry.

More recently, it has become clear that these models are only the simplest representatives of a much larger class of models. For concreteness, we will restrict our discussion to the type IIB theory, but similar remarks would apply in the other limits of M-theory. In the IIB context, the theory contains p-form gauge fields which couple to the various supersymmetric D-branes and NS-branes. One can consider magnetic fluxes of these gauge fields in the internal dimensions; such fluxes are consistent with 4d Lorentz invariance. In fact, in type II orientifold models which break the supersymmetry from $\mathcal{N}=2$ to $\mathcal{N}=1$, generic methods of satisfying the tadpole conditions will involve the addition of background magnetic flux. This is interesting because the magnetic fluxes will cost an energy which depends on the detailed shape and size of the internal dimensions; that is, they will generate a potential for some of the Calabi-Yau moduli.

In the case of type IIB on a Calabi-Yau orientifold, the allowed fluxes are those of the RR and NS three-form field strengths F_3 and H_3. The properties of the theory in the presence of such fluxes have been investigated in a vast literature; see for instance [2]–[7]. The resulting $\mathcal{N} = 1$ supergravity theories (derived in [4, 8]) are of the no-scale type, with a Kähler potential K inherited (at large volume and weak coupling) from the Calabi-Yau compactification and a superpotential which is computable in terms of the choice of quantized background magnetic fluxes [3]. The resulting scalar potential generically fixes the complex structure moduli and the dilaton. It is possible to make semi-realistic chiral models of particle physics in this framework [9], and to describe some aspects of the resulting supergravity theories in the general framework of gauged extended supergravity [10]. The theory of soft susy breaking terms in these models has been developed in [11].

For a given choice of the integral background fluxes, the superpotential for moduli can simply be written as a combination of periods of the holomorphic three-form Ω on the Calabi-Yau space. The periods are computable by standard techniques in classical geometry, highly developed in the physics literature after the seminal work of Candelas et al on mirror symmetry [12]. The resulting superpotential is exact at least to all orders in the α' expansion by standard holomorphy arguments. It can be argued that "generic" non-perturbative effects (known to occur in various models in various circumstances, and likely to occur in a generic setting due to the absence of symmetries that would prevent them) should suffice to fix the remaining (Kähler) moduli in some cases [13, 14]. As argued in [13], if the SUSY-breaking ($e^K|W|^2$ evaluated in vacuum) by the fluxes is suitably small, and if g_s is stabilized by the fluxes at weak coupling, one can have small expansion parameters which allow one to stabilize the remaining moduli in a regime of control. Of course, this will only happen in at most a small fraction of the vacua. As emphasized in [13], following the basic ideas of Bousso and Polchinski [15] (see also [16]), the large number of possible integral flux choices makes it plausible that suitable flux choices do exist, which allow one to fix g_s at weak coupling and keep the SUSY breaking hierarchically smaller than the string scale.

This set of issues, which goes into establishing the existence of a "landscape" of string/M-theory vacua (like the "discretuum" of [15]) and characterizing its properties, can and should be studied systematically and quantitatively. In addition to the works already described, a persuasive discussion of this can be found in the works of Douglas and collaborators [14] and [17]–[19] (see also [20, 21] for more general comments about the landscape). In particular, in [17], under well motivated assumptions some theorems are proven regarding the numbers and distribution of flux vacua on the Calabi-Yau moduli space (really, the moduli space of complex structures and dilaton VEVs) $\mathcal{M}$. The assumption which goes into proving these theorems is, roughly, that the superpotential is a random section (with respect to a covariance kernel determined by the Kähler potential) of the appropriate line bundle over $\mathcal{M}$. This models the flux superpotentials as random

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combinations of periods with respect to some measure, instead of as the precise integral combinations of periods determined by various choices of H_3 and F_3 consistent with the tadpole conditions. Basically, this assumption follows from the statement that if one allows large enough fluxes, flux quantization becomes irrelevant for the average properties of the vacua. The natural size of the fluxes required by tadpole conditions in IIB string theory probably falls into this range.

Because the emerging discretuum of vacua is too complicated to allow us to usefully examine each vacuum individually, this kind of program to find simple assumptions which accurately characterize the space of vacua seems to us rather important. In fact, following [17, 19], one could simply disregard the Kähler moduli altogether, ignore the no-scale structure (which is certainly broken by quantum corrections), and characterize the properties of the supergravities that result with the given K and W as a function of complex structure moduli. The resulting ensemble of supergravity theories is rich enough that it is very plausible that some aspects of the full story (about the real space of string vacua), are captured by this simplified ensemble. However, we will continue to use the language and equations appropriate to thinking of these as vacua which arise in the no-scale approximation to the full IIB theory, though it is easy to translate back and forth.

In this paper, we test the prediction of [17] regarding the distribution of vacua on $\mathcal{M}$, by studying the vacua which result on a subspace of the moduli space of a certain Calabi-Yau threefold, in the presence of a small subset of the possible fluxes. While we are looking at a very small fraction of the possible fluxes and hence of the total number of flux vacua on this space, the numbers even on our subspace are easily large enough to allow a meaningful test of the conjecture. In addition, in these vacua there is a tadpole condition that requires $\int H_3 \wedge F_3 = N_{\mathrm{flux}} \leq L$, where L is some number fixed by global details of the model. The growth of the number of vacua with L, fixing the number of cycles which support flux, is subject to various conjectures along the lines of [15]. Here, in addition to testing the distribution of vacua on moduli space, we also test the growth of the number of vacua as a function of L, and find a simple power law. A proper understanding of the theory which determines this power is rather complicated, as we describe in the following; some relevant results are discussed in the revised version of [19].

The organization of this paper is as follows. In section 2, we describe the predictions we wish to test. In section 3, we discuss the model in which we will test them (which is "model A" of [7]). This model has (on the locus of interest to us) a one-dimensional complex structure moduli space, parametrized by a complex variable ψ. In sections 4 and 5, we describe how we find solutions of the flux equations in two regions in the moduli space, using expansions of the periods around $\psi = 0$ (the mirror Landau-Ginzburg point) and $\psi = 1$ (the conifold point). In each of these cases, we find sufficiently large numbers of vacua to perform meaningful tests of the predictions. In section 6 we present our conclusions.

We should stress that although the discussion here is specific to type IIB vacua of the class described in [4] (for which several de Sitter scenarios exist [13, 22]), a similar landscape also emerges from careful consideration of the various other limits of M-theory (see e.g. [23] for discussions in the heterotic setting, and [24] for an early construction of de Sitter vacua in noncritical string theories). For instance in the heterotic string, one needs to consider

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the large class of possible gauge bundles in each E_8, in addition to the possibility of NS three-form flux and Non-Kählerity [25]. This is technically more challenging than studying the class of IIB flux vacua we considered here, but finding similar tractable toy ensembles in that setting should be possible. Another setting where similar equations arise is in the problem of finding BPS black holes in $\mathcal{N} = 2$ supergravity; the interesting connections between that problem and the problem of finding flux vacua are described in [26].

While this work was being completed, the paper [19] which has some overlap with our results appeared.

2. The conjectures

We will be interested in testing two conjectures about the large class of IIB flux vacua described in [4], in the no-scale approximation. The conjectures concern the distribution of flux vacua on the moduli space of complex structures, and the scaling of the number of flux vacua with the D3-charge in the fluxes. They can be proven to hold [17] under suitable assumptions about the irrelevance of flux quantization, so we are really testing whether those assumptions hold for simple representatives of the class of vacua described in [4].

2.1 Where do the vacua lie?

In a recent paper of Ashok and Douglas [17], it was conjectured that the index density of the no-scale vacua described in [4] is given by an integral

$$\mathcal{I} \sim \int_{\mathcal{M}} \det(-R - \omega)\,. \tag{2.1}$$

$\mathcal{I}$ is normalized appropriately to the total number (or index) of no-scale vacua. Here ω is the Kähler form on the moduli space and R is the curvature two-form expressed as a hermitean matrix

$$R^l_k = R^l_{i\bar{j}k} dz^i \wedge d\bar{z}^{\bar{j}} \tag{2.2}$$

with z^i the coordinates on $\mathcal{M}$. The moduli space $\mathcal{M}$ consists of the fundamental domain of any relevant duality groups, acting on the complex structure and axio-dilaton Teichmuller spaces. In our example, it will be a product of the standard fundamental domain of $SL(2, Z)$ in the upper half-plane (for the axio-dilaton τ) with a (complex) one-dimensional slice of a Calabi-Yau complex structure moduli space. As described in [17, 19] (see especially [19, section 3.1.2]), one can integrate over τ-space and find a distribution of vacua on just the complex structure moduli space; for one-modulus models one finds

$$\mathcal{I} \sim \int_{\mathcal{M}} c_1\,, \tag{2.3}$$

where now $\mathcal{M}$ is just the complex structure moduli space, and c_1 is the first Chern class.

We will assume that there are no miraculous cancellations, so that the total number of vacua scales like $\mathcal{I}$. In this case, one predicts that the number of vacua in a given region of complex structure moduli space should scale as in (2.3). We will test this conjecture in sections 4 and 5 by computing the numbers of vacua lying in various regions of the

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moduli space (by using Monte Carlo simulations to choose fluxes and then solving the flux equations), and checking that they are distributed like the appropriate integrals of the first Chern class.

2.2 Growth of number of vacua with L

The three-form fluxes in type IIB supergravity contribute to the D3-brane charge tadpole on the compact space, via an equation of the form

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2}\int_M H_3 \wedge F_3\,, \tag{2.4}$$

where tadpole cancellation requires

$$N_{\text{D3}} + N_{\text{flux}} = L\,. \tag{2.5}$$

Here L is some number fixed by the orientifold charge in the given model and N_{D3} is the number of space-time filling D3 branes, transverse to M.

While it may appear from (2.5) that this is not much of a restriction, since the quadratic form $\int H_3 \wedge F_3$ is not positive definite, this is not quite correct. On solutions of the equations of motion, one finds (as on p. 4 of the first reference in [5]) that $F_3 \sim - *_6 H_3$, so N_{flux} becomes a positive semidefinite form. This means that in a space with K three-cycles, one might heuristically think of (2.5) as allowing one to search for flux vacua at each integral lattice point within a sphere of radius $\sqrt{L}$ in flux space. Although H and F together involve 2K flux integers, using the relation between H and F on solutions indicates that for a fixed point in moduli space, this will be a $K-1$ sphere (instead of a $2K-1$ sphere). So for e.g. a $(T^2)^3$ with square two-tori, one can easily rewrite (2.5) (setting $N_{D3}=0$) as the equation for a sphere in K dimensions. Then, the number of solutions for the flux equations N_{vacua}, like the number of lattice points inside this sphere, presumably scales like

$$N_{\text{vacua}} \sim L^{K/2}\,. \tag{2.6}$$

This would be the behavior of the number of solutions, in the simple toroidal example above, at a point in $\mathcal{M}$ where solutions to the flux equations exist.[1]

This is the kind of logic suggested in e.g. [15], applied in this context by many workers [28], and rigorously argued to hold in this context in [17]. However, the results of [17] contain a surprise – the predicted scaling of the total number of vacua is actually

$$N_{\text{vacua}} \sim L^K \tag{2.7}$$

instead of the $L^{K/2}$ in (2.6). To understand this,[2] one must carefully consider a smoothed average of the number of solutions per unit volume in $\mathcal{M}$ — the number of solutions jumps discontinuously as one moves in $\mathcal{M}$, so one must average to get a reasonable formula. This

[1] One might ask whether one could increase the number of solutions by including anti-D3 branes (i.e. taking $N_{D3} < 0$ in (2.5)). While one can include anti-branes in these models, the number is bounded by the decay mechanism discussed in [27], and we will simply assume $N_{D3} \geq 0$ for now.

[2] We are grateful to M. Douglas for several discussions about this subject.

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averaging produces an enhancement of the result – for large enough L, a given small region of moduli space will give rise to L^K lattice points within flux space, instead of the naive $L^{K/2}$. This in particular means that if one looks at finite L in some region of the moduli space, one may find an intermediate scaling that is between the naive prediction (2.6) and the L^K behavior (2.7) expected from [17]. The detailed crossover behavior determining how the scaling should go for a given region in $\mathcal{M}$ at various L and K is a complicated subject which is still under investigation (see the revised version of the paper by Denef and Douglas [19] for further discussion).

Because of the complications described above, we simply take our results as an experimental clue that should help in the development of the full theory of how N_{vacua} scales with L and K, though we expect to (and do) find a power intermediate between (2.6) and (2.7).

3. The model of interest

We will study an orientifold of the Calabi-Yau threefold M which arises as a hypersurface in the weighted projective space $WP^4_{1,1,1,1,4}$

$$4x_0^2 + x_1^8 + x_2^8 + x_3^8 + x_4^8 - 8\psi x_0 x_1 x_2 x_3 x_4 = 0\,. \tag{3.1}$$

This threefold has $h^{1,1}(M) = 1$ and $h^{2,1}(M) = 149$. As explained in [7], an orientifold of M arises as a limit of the F-theory compactification on the Calabi-Yau fourfold X given by a hypersurface in $WP^5_{1,1,1,1,8,12}$. The orientifold action takes $x_0 \to -x_0$ and $\psi \to -\psi$ (in conjunction with the worldsheet parity reversal), while leaving all other variables unchanged. This limit of the F-theory model on X is obtained by following the general prescription given by Sen [29]. One of the benefits of knowing the F-theory lift is that it allows us to easily compute the tadpole condition for compactification on the orientifold of M. The relevant condition is (see [30] and the third paper listed in [2])

$$N_{\mathrm{D3}} + N_{\mathrm{flux}} = \frac{\chi(X)}{24} = 972\,, \tag{3.2}$$

where N_{D3} is the number of space-filling D3 branes one inserts transverse to M, and N_{flux} is the amount of D3 charge carried by the H_3 and F_3 fluxes as given in (2.4).

In general, one could add many deformations to the defining equation (3.1) which are consistent with the orientifold action. However, as explained in [7] following [31], the equation (3.1) is the most general equation consistent with a global symmetry group $\mathcal{G} = Z_8^2 \times Z_2$ acting on the projective coordinates. If one turns on only fluxes which are consistent with preserving this symmetry, then the resulting superpotential is guaranteed to depend on the additional moduli only at higher orders (in $\mathcal{G}$ invariant combinations). This means that if we set these moduli to zero (as done in (3.1)), compute the periods relevant for our subset of fluxes, and solve the resulting equations for ψ and the axio-dilaton τ, we are guaranteed that the solutions we find will be solutions of the full theory. The additional moduli can be consistently set to zero because of the symmetry, and will generically be constrained by a higher order potential in the resulting vacua.

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In fact, the subset of periods which are relevant to the $\mathcal{G}$ invariant fluxes coincide (up to a factor of $|\mathcal{G}|$) with those of the mirror manifold to M, W, as explained in [7]. These correspond to periods on four different 3-cycles in M, and hence we proceed to develop the relevant formalism for computing flux superpotentials involving only H_3 and F_3 fluxes through these four 3-cycles. Happily, the periods of interest to us in this example were computed in one of the early studies of mirror symmetry, by Klemm and Theisen [32].

In comparing to the conjecture of section 2.2, we see that our problem has $K = 4$ and $L = 972$. So in the physical problem of interest, we could check the number of vacua with flux charge $N_{\rm flux} \leq L_*$ for any $L_* \leq 972$. (Of course formally, we are free to consider vacua using the periods from this model with arbitrary L).

3.1 Homology and Cohomology bases

We will work with a symplectic homology basis for the subspaces of H_3 of interest to us. The basis of three-cycles A_a and B^a $(a = 1, 2)$ and the basis for integral cohomology α_a and β^a satisfy

$$\int_{A_a} \alpha_b = \delta^a_b \,, \qquad \int_{B^b} \beta^a = -\delta^a_b \,, \qquad \int_M \alpha_a \wedge \beta^b = \delta^b_a \,. \tag{3.3}$$

The holomorphic three form can be represented in terms of periods in this basis as follows:

$$\int_{A_a} \Omega = z^a \,, \qquad \int_{B^a} \Omega = \mathcal{G}_a \,, \qquad \Omega = z^a \alpha_a - \mathcal{G}_a \beta^a \,. \tag{3.4}$$

In addition

$$\int_M \Omega \wedge \overline{\Omega} = \overline{z}^a \mathcal{G}_a - z^a \overline{\mathcal{G}}_a = \overline{z}^a \frac{\partial \mathcal{G}}{\partial z^a} - z^a \frac{\partial \overline{\mathcal{G}}}{\partial \overline{z}^a} = -\Pi^\dagger \cdot \Sigma \cdot \Pi \,. \tag{3.5}$$

Here, we have introduced the prepotential $\mathcal{G}(z^1, z^2)$, the period vector Π (whose entries are the periods (3.4)), and the matrix

$$\Sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{3.6}$$

whose entries are two by two matrices.

3.2 Fluxes, superpotential and Kähler potential

The NS and RR fluxes admit the following quantization condition

$$F_{(3)} = (2\pi)^2 \alpha' (f_a [B_a] + f_{a+2} [A_a]) \,, \qquad H_{(3)} = (2\pi)^2 \alpha' (h_a [B_a] + h_{a+2} [A_a]) \tag{3.7}$$

with integer f_i and h_i. Here we used the notation $[A_a] = \alpha_a$ and $[B_a] = \beta_a$. Using this notation, we find the following expression for $N_{\rm flux}$

$$N_{\rm flux} = \frac{1}{(2\pi)^4 (\alpha')^2} \int_M H_{(3)} \wedge F_{(3)} = f^T \cdot \Sigma \cdot h \,. \tag{3.8}$$

The superpotential is given by

$$W = \int_M (F_{(3)} - \tau H_{(3)}) \wedge \Omega = (2\pi)^2 \alpha' (f \cdot \Pi - \tau h \cdot \Pi) \,. \tag{3.9}$$

The Kähler potential for the dilaton-axion and complex moduli is given by

$$K = K_\tau + K_\psi = -\ln(-i(\tau-\overline{\tau})) - \ln(i\int_M \Omega\wedge\overline{\Omega}) = -\ln(-i(\tau-\overline{\tau})) - \ln(-i\Pi^\dagger\cdot\Sigma\cdot\Pi)\,. \quad (3.10)$$

3.3 Conditions for solutions

The tadpole condition for the model of interest is given by (3.2). This provides a restriction on the fluxes we can use, i.e. on our allowed class of superpotentials — we must keep $N_{\rm flux} \leq 972$.

We will be searching for nonsupersymmetric flux vacua which obey the equations

$$D_\tau W = 0 \qquad \text{and} \qquad D_\psi W = 0\,. \quad (3.11)$$

They are nonsupersymmetric because in the full IIB string theory, they will generically have $D_\rho W \sim W \neq 0$ where ρ is a Kähler modulus. Note however that in the toy ensemble of[17, 19], such vacua are called supersymmetric, since the Kähler moduli are not part of the toy ensemble.

4. Small ψ region

4.1 Periods, Kähler potential, metric and curvature form

The periods for the small ψ, $|\psi| < 1$ region are given by

$$w_j(\psi) = (2\pi i)^3\frac{\pi}{8}\sum_{n=1}^{\infty}\frac{1}{\Gamma(n)(\Gamma(1-\frac{n}{8}))^4\Gamma(1-\frac{n}{2})}\frac{\exp\left(\frac{7\pi i}{8}n\right)}{\sin\left(\frac{\pi n}{8}\right)}\frac{(4\alpha^j\psi)^n}{\psi}\,, \quad (4.1)$$

where $\alpha = \exp(\pi i/4)$. The corresponding period vector is given by

$$w^T = (w_2(\psi), w_1(\psi), w_0(\psi), w_7(\psi))\,. \quad (4.2)$$

We will want the periods in a symplectic basis. In such a basis, the period vector $\Pi^T(\psi) = (\mathcal{G}_1(\psi), \mathcal{G}_2(\psi), z^1(\psi), z^2(\psi))$ is provided by the linear transformation

$$\Pi \;=\; m\cdot w\,, \quad (4.3)$$

where the matrix m is given by

$$m = \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 3 & 2 \\ 0 & 1 & -1 & 0 \end{pmatrix}. \quad (4.4)$$

The period vector in the symplectic basis can be written in the form

$$\Pi(\psi) = \sum_{n=0}^{\infty} c_{2n}p_{2n}\psi^{2n}\,. \quad (4.5)$$

Here the vectors p_{2n} are given by

$$p_{2n} = \ m \cdot (\alpha^{2(2n+1)}, \alpha^{(2n+1)}, 1, \alpha^{7(2n+1)})^T \tag{4.6}$$

and the first few constants c_{2n} of interest are given by

$$\begin{aligned}
c_0 &= (2\pi i)^3 \frac{\sqrt{\pi}}{2\Gamma^4(7/8)} \frac{\exp(\frac{7\pi i}{8})}{\sin(\frac{\pi}{8})}\,, & c_2 &= -(2\pi i)^3 \frac{2\sqrt{\pi}}{\Gamma^4(5/8)} \frac{\exp(\frac{5\pi i}{8})}{\sin(\frac{3\pi}{8})}\,, \\
c_4 &= (2\pi i)^3 \frac{4\sqrt{\pi}}{\Gamma^4(3/8)} \frac{\exp(\frac{3\pi i}{8})}{\sin(\frac{5\pi}{8})}\,, & c_8 &= (2\pi i)^3 \frac{\sqrt{\pi}}{768\Gamma^4(7/8)} \frac{\exp(\frac{7\pi i}{8})}{\sin(\frac{\pi}{8})}\,.
\end{aligned} \tag{4.7}$$

This yields the following Kähler potential for the complex structure moduli space:

$$K_\psi = -\ln[\lambda_0 + \lambda_2 |\psi|^4 + \lambda_4 |\psi|^8 + \lambda_5 \psi^8 + \overline{\lambda}_5 \overline{\psi}^8 + O(|\psi|^{12})]\,, \tag{4.8}$$

where the real constants are given by

$$\begin{aligned}
\lambda_0 &= -i|c_0|^2 p_0^\dagger \cdot \Sigma \cdot p_0\,, & \lambda_2 &= -i|c_2|^2 p_2^\dagger \cdot \Sigma \cdot p_2\,, \\
\lambda_4 &= -i|c_4|^2 p_4^\dagger \cdot \Sigma \cdot p_4\,, & \lambda_5 &= -i\overline{c}_0 c_8 p_0^\dagger \cdot \Sigma \cdot p_0\,.
\end{aligned} \tag{4.9}$$

Note that $p_0^\dagger \cdot \Sigma \cdot p_0 = 2i(\sqrt{2}+2)$ and $p_2^\dagger \cdot \Sigma \cdot p_2 = -p_4^\dagger \cdot \Sigma \cdot p_4 = 2i(\sqrt{2}-2)$.

The resulting Kähler metric takes the form

$$g_{\psi\overline{\psi}} = -4\frac{\lambda_2}{\lambda_0}|\psi|^2 \left(1 + \left(4\frac{\lambda_4}{\lambda_2} - 2\frac{\lambda_2}{\lambda_0}\right)|\psi|^4 + O(\psi^8)\right). \tag{4.10}$$

Using the expression for the curvature form

$$R_{\psi\overline{\psi}} = R^{\psi}_{\psi\overline{\psi}\psi} = -\partial_\psi \partial_{\overline{\psi}} \ln(g_{\psi\overline{\psi}}) \tag{4.11}$$

one finds the following expression of interest

$$R_{\psi\overline{\psi}} = -4|\psi|^2 \left(4\frac{\lambda_4}{\lambda_2} - 2\frac{\lambda_2}{\lambda_0}\right) + O(|\psi|^6)\,. \tag{4.12}$$

Note also that one finds

$$\partial_\psi K_\psi = -2\frac{\lambda_2}{\lambda_0}|\psi|^2 \overline{\psi} + O(\psi^7)\,, \tag{4.13}$$

which is of use in evaluating Kähler covariantized derivatives with respect to ψ.

4.2 Distribution of flux vacua

Given these facts about the Calabi-Yau moduli space geometry, it is now possible to work out the detailed prediction from §2.1. One sees that

$$\int_{\mathcal{M}} 2\pi i\, c_1(\psi) = \text{const} \int_0^r |\psi|^2 |\psi| d|\psi| = \text{const}\frac{r^4}{4} \tag{4.14}$$

for (an eight-fold cover of) a small piece of the fundamental region of complex structure moduli space $\mathcal{M}$: $|\psi| < r$.

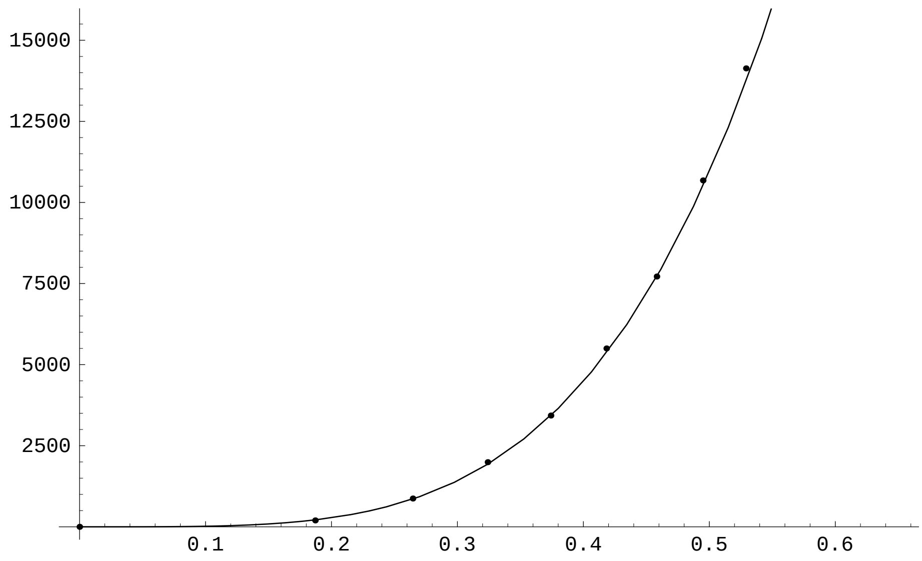

Figure 1: A plot of a numerical evaluation of $\int_{\mathcal{M}} 2\pi i\, c_1(\psi)$. The Monte Carlo simulation data for each point is: number of random fluxes $N = 10^8$; random flux interval $f, h \in (-40, 40)$. The data is fit by the curve $700000\,\frac{r^4}{4}$.

The equations for flux vacua, keeping terms up to order ψ^2, become

$$\begin{aligned} D_\tau W &= 0 \;\Rightarrow\; (f - \overline{\tau} h)\cdot(c_0 p_0 + c_2 p_2 \psi^2) = 0\,; \\ D_\psi W &= 0 \;\Rightarrow\; (f - \tau h)\cdot\left(c_2 p_2 + 2 c_4 p_4 \psi^2 - \frac{\lambda_2}{\lambda_0} c_0 p_0 \overline{\psi}^2\right) = 0\,. \end{aligned} \tag{4.15}$$

This provides two equations

$$\tau = \frac{f\cdot p_0^\dagger + \frac{\overline{c}_2}{\overline{c}_0} f\cdot p_2^\dagger \overline{\psi}^2}{h\cdot p_0^\dagger + \frac{\overline{c}_2}{\overline{c}_0} h\cdot p_2^\dagger \overline{\psi}^2} = \frac{f\cdot p_2 + \frac{2c_4}{c_2} f\cdot p_4 \psi^2 - \frac{\lambda_2 c_0}{\lambda_0 c_2} f\cdot p_0\, \overline{\psi}^2}{h\cdot p_2 + \frac{2c_4}{c_2} h\cdot p_4 \psi^2 - \frac{\lambda_2 c_0}{\lambda_0 c_2} h\cdot p_0\, \overline{\psi}^2}\,. \tag{4.16}$$

The last one gives a linear equation for ψ^2 such that

$$a + b_1 \psi^2 + b_2 \overline{\psi}^2 + O(\psi^4) = 0\,, \tag{4.17}$$

where we defined the following quantities depending on fluxes

$$\begin{aligned} a &= f\cdot p_0^\dagger\, h\cdot p_2 - h\cdot p_0^\dagger\, f\cdot p_2\,; \\ b_1 &= \frac{2c_4}{c_2}(h\cdot p_4\, f\cdot p_0^\dagger - f\cdot p_4\, h\cdot p_0^\dagger)\,; \\ b_2 &= \frac{\lambda_2 c_0}{\lambda_0 c_2}(f\cdot p_0\, h\cdot p_0^\dagger - h\cdot p_0\, f\cdot p_0^\dagger) + \frac{\overline{c}_2}{\overline{c}_0}(f\cdot p_2^\dagger\, h\cdot p_2 - h\cdot p_2^\dagger\, f\cdot p_2)\,. \end{aligned} \tag{4.18}$$

Introducing the notation $\psi^2 \equiv x + iy$ one gets the following system of linear equations

$$\begin{aligned} \mathrm{Re}(a) + \mathrm{Re}(b_1 + b_2)x - \mathrm{Im}(b_1 - b_2)y &= 0\,; \\ \mathrm{Im}(a) + \mathrm{Im}(b_1 + b_2)x + \mathrm{Re}(b_1 - b_2)y &= 0\,. \end{aligned} \tag{4.19}$$

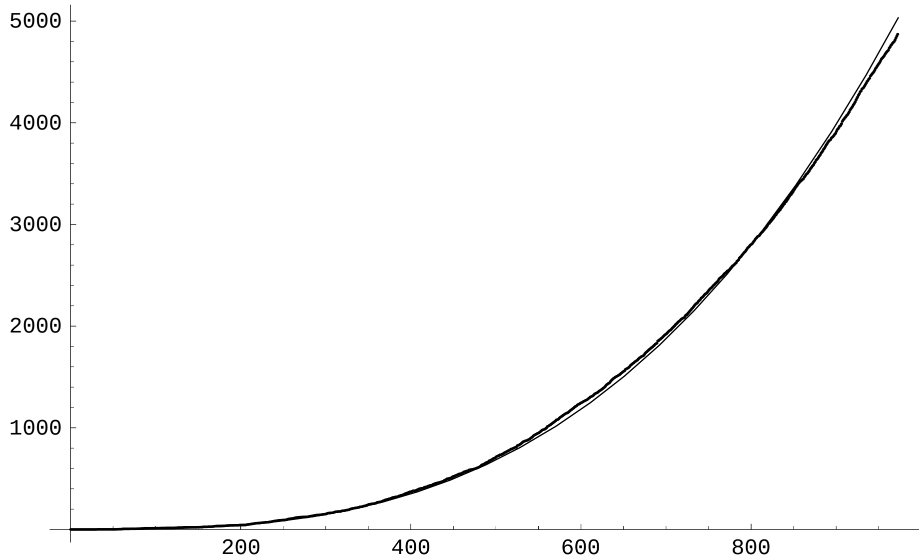

Figure 2: The numerical results for the number of vacua with $N_{\text{flux}} < L$ for $L \in (1, 972)$. The Monte Carlo simulation data is: number of random fluxes $N = 10^9$; random flux interval $f, h \in (-100, 100)$; complex structure ψ space region $|\psi|^2 < 0.5$. The data is fit by the curve $\frac{L^3}{183000}$.

We used Monte Carlo simulations to check the validity of the formula (4.14) and the scaling of the number of vacua with L. We chose correspondingly 10^8 and 10^9 random fluxes, taking integer values f, h in the ranges indicated in the figure captions. The number of solutions with $L \leq 972$ resulting in small ψ is determined numerically from eqs. (4.16)–(4.19) and is plotted in figure 1. Figure 2 shows the dependence of the total number of vacua on L.

One sees that the curve in figure 1 is in very good agreement with the prediction (2.1). It is interesting that in figure 2, one sees a rather clean L^3 growth of the number of vacua. The naive formula (2.6) would predict L^2, while the more refined formula proved by Ashok and Douglas under certain assumptions would predict L^4. In fact, as described in section 2.2 and the revised version of [19], the correct scaling for a small region in ψ-space is a complicated subject. It may be that exceedingly large L would be required to attain the scaling (2.7), in this region of moduli space.

5. Conifold region

5.1 Periods, Kähler potential, metric and curvature form

The periods in a symplectic basis in the vicinity of the conifold point $\psi = 1$ can be given to the first order by the following expressions (here $x \equiv 1 - \psi$ and $|x| \ll 1$)

$$
\begin{aligned}
\mathcal{G}_1(x) &= (2\pi i)^3 [a_0 + a_1 x + O(x^2)] \,, \\
\mathcal{G}_2(x) &= \frac{z^2(x)}{2\pi i} \ln(x) + (2\pi i)^3 [b_0 + b_1 x + O(x^2)] \,, \\
z^1(x) &= (2\pi i)^3 [c_0 + c_1 x + O(x^2)] \,, \\
z^2(x) &= (2\pi i)^3 [d_0 + d_1 x + O(x^2)] \,,
\end{aligned} \tag{5.1}
$$

where the constants can be approximated by the following numbers

$$\begin{aligned} a_0 &= -1.774i\,, & c_0 &= 4.952 - 5.321i\,, \\ a_1 &= 1.227i\,, & c_1 &= -4.488 + 3.682i\,, \\ b_0 &= -1.047\,, & d_0 &= 0\,, \\ b_1 &= 0.451 + 0.900i\,, & d_1 &= 1.800i\,. \end{aligned} \tag{5.2}$$

The Kähler potential for complex structure modulus is given by

$$K_\psi = -\ln[\mu_0 + \mu_1 x + \overline{\mu}_1\overline{x} + \mu_2|x|^2\ln|x|^2 + \mu_3|x|^2 + \mu_4 x^2 + \overline{\mu}_4\overline{x}^2 + O(|x|^3\ln|x|)]\,, \tag{5.3}$$

where the relevant constants μ_0, μ_1, μ_2 and μ_3 are given by

$$\begin{aligned} \mu_0 &= i(2\pi)^6(a_0\overline{c}_0 - c_0\overline{a}_0)\,, & \mu_1 &= i(2\pi)^6(\overline{c}_0 a_1 - c_1\overline{a}_0 - d_1\overline{b}_0)\,, \\ \mu_2 &= (2\pi)^5|d_1|^2\,, & \mu_3 &= i(2\pi)^6(\overline{c}_1 a_1 - \overline{a}_1 c_1 + \overline{d}_1 b_1 - \overline{b}_1 d_1)\,. \end{aligned} \tag{5.4}$$

One finds the following expression for the Kähler metric

$$g_{x\overline{x}} = -\frac{\mu_2}{\mu_0}\ln|x|^2 + \left(\frac{|\mu_1|^2}{\mu_0^2} - \frac{2\mu_2+\mu_3}{\mu_0}\right) + O(|x|\ln|x|)\,. \tag{5.5}$$

Then the curvature form is

$$R_{x\overline{x}} = \frac{1}{4|x|^2}\frac{1}{(\ln|x|+C)^2}\,, \tag{5.6}$$

where the constant C is determined to be

$$C = 1 - \frac{|\mu_1|^2}{2\mu_0\mu_2} + \frac{\mu_3}{2\mu_2} \approx -0.738\,. \tag{5.7}$$

In computing Kähler covariantized derivatives with respect to ψ, it is also useful to note that

$$\partial_x K_\psi = -\frac{\mu_1}{\mu_0} - \frac{\mu_2}{\mu_0}\overline{x}\ln|x|^2 + O(x)\,. \tag{5.8}$$

5.2 Distribution of flux vacua

We numerically solved for vacua around $\psi = 1$. To compare with the prediction from section 2.1, we need to determine the curvature and Kähler form of the moduli space. Using the results of section 5.1, we find (including only the leading contribution coming from the curvature form)

$$\int_{\mathcal{M}} 2\pi i\, c_1(\psi) = \text{const}\int_0^r \frac{1}{|x|^2}\frac{1}{(\ln|x|+C)^2}|x|d|x| = -\frac{\text{const}}{\ln r + C} \tag{5.9}$$

for the portion of the fundamental region of complex structure moduli space $\mathcal{M}$: $|x| < r$.

The equations for flux vacua take the form

$$D_\tau W = 0 \quad\Rightarrow\quad \tau = \frac{f\cdot\Pi^\dagger}{h\cdot\Pi^\dagger} = \frac{f_1\overline{a}_0 + f_2\overline{b}_0 + f_3\overline{c}_0}{h_1\overline{a}_0 + h_2\overline{b}_0 + h_3\overline{c}_0} + O(|x|\ln|x|)\,; \tag{5.10}$$

$$\begin{aligned} D_\psi W = 0 \quad\Rightarrow\quad \ln(x) = -\frac{2\pi i}{d_1}\Bigg[&\frac{(f_1-\tau h_1)(a_1 - \frac{\mu_1}{\mu_0}a_0) + (f_2-\tau h_2)(b_1 - \frac{\mu_1}{\mu_0}b_0)}{f_2-\tau h_2} + \\ &+ \frac{(f_3-\tau h_3)(c_1 - \frac{\mu_1}{\mu_0}c_0) + (f_4-\tau h_4)d_1}{f_2-\tau h_2}\Bigg] - 1\,. \end{aligned} \tag{5.11}$$

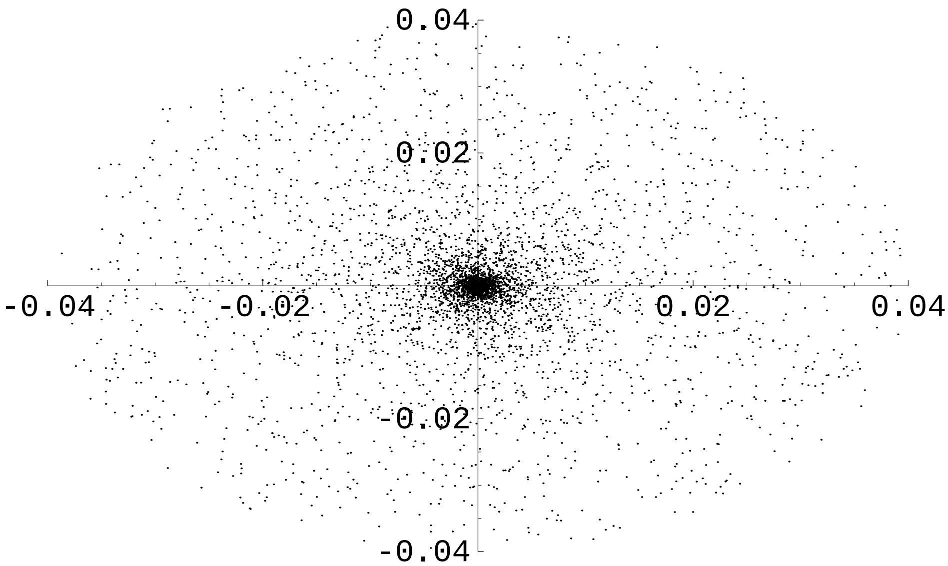

Figure 3: Each point is a vacuum on the $x = 1 - \psi$ complex plane. The Monte Carlo simulation data is: number of random fluxes $N = 5 \times 10^7$; random flux interval $f, h \in (-100, 100)$; complex structure ψ space region $|x| < 0.04$. There are 11249 vacua, but 6306 of them arise at $|x| < .00001$ and have been removed from the plot (they would all cluster at the origin).

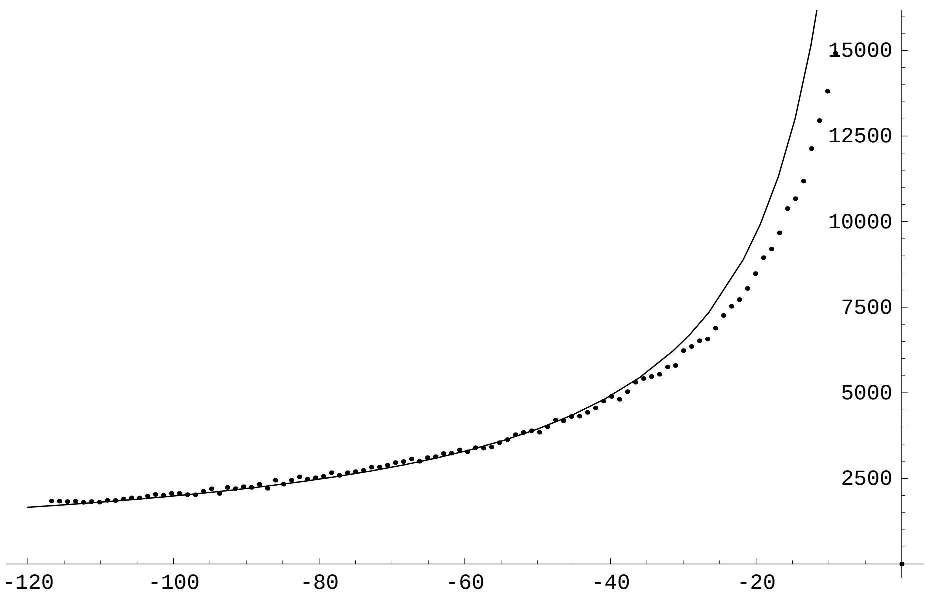

Figure 4: The plot of a numerical evaluation of $\int_{\mathcal{M}} 2\pi i\, c_1(\psi)$. The Monte Carlo simulation data for each point is: number of random fluxes $N = 10^7$; random flux interval $f, h \in (-60, 60)$. The data is fit by the curve $-\frac{200000}{\rho+C}$, where $\rho = \ln r$. The conifold point $r = 0$ is at $\rho = -\infty$ for this coordinate.

We performed a Monte Carlo simulation, solving the equations of motion for randomly selected fluxes. In figure 3, we plot the distribution of vacua near $\psi = 1$ on the $x = 1 - \psi$ plane. The most striking thing about figure 3 is the clustering of vacua around the conifold

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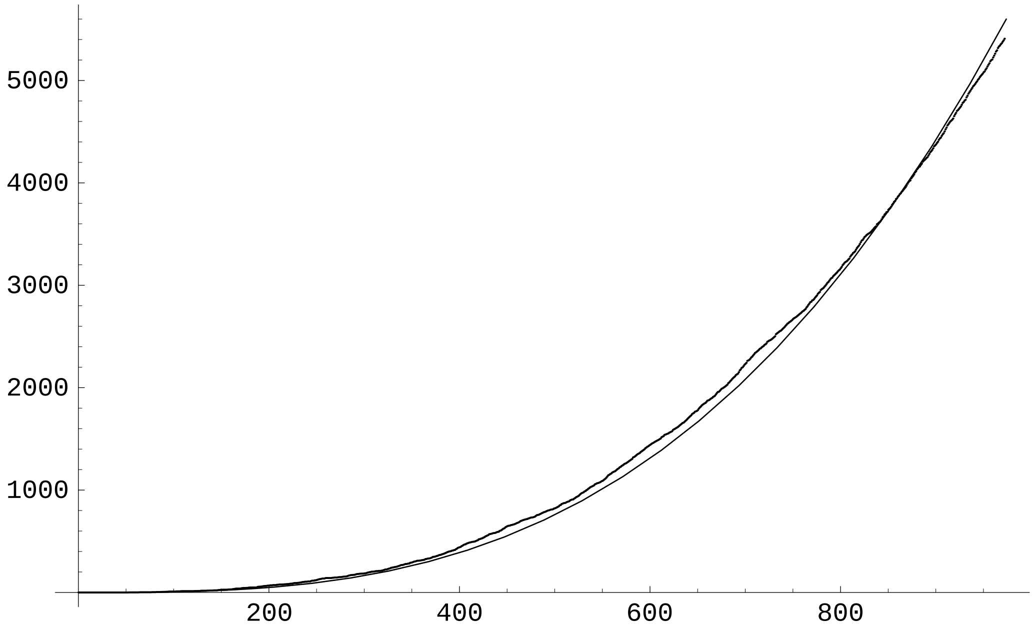

Figure 5: The numerical results for the number of vacua with $N_{\text{flux}} < L$ for $L \in (1, 972)$. The Monte Carlo simulation data is: number of random fluxes $N = 10^{10}$; random flux interval $f, h \in (-200, 200)$; complex structure ψ space region $|x| < 0.001$. The data is fit by the curve $\frac{L^3}{165000}$.

point [19, 33]. This would be expected from the curvature dependence of the result for the index (2.1) (which, as shown in figure 4, is in good agreement with our results). This is interesting both because it shows that the conjecture (2.1) contains much more information than a plausible boring conjecture like "vacua are uniformly distributed on the moduli space," and that furthermore the detailed distribution we find experimentally is in fact captured by the simple formula (2.1)!

As described in [4, 13], the conifold point is particularly interesting because it provides a natural mechanism for flux-generated hierarchies (as in the noncompact Klebanov-Strassler solution [34]), and can provide small numbers which help with moduli stabilization or attaining small vacuum energy. It could therefore also be physically important that it dominates the distribution in figure 3. Flux vacua near the conifold [4] (see also [35]) can explain light scalars by warping (instead of protecting their masses by supersymmetry). Therefore, if this feature persisted in the full problem of characterizing the string landscape, it could mean that the correct stringy notion of naturalness allows warping as a natural explanation for light scalars (as in Randall-Sundrum scenarios [36]), in addition to low-energy supersymmetry. Of course this is mere speculation at this point, but indicates the kinds of lessons one could hope to learn.

As shown in figure 5, for the range of L most relevant to Model A of [7], we again seem to obtain close to L^3 scaling for the number of vacua in this region of moduli space. It is quite possible that at much larger L, this behavior would be modified to L^4 in accord with (2.7), and that a more detailed theory could explain the L^3 scaling visible here.

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6. Conclusions

In this paper, we have seen that very simple conjectures can serve to capture the detailed structure of the numbers and distribution of flux vacua on Calabi-Yau moduli spaces. While the flux potentials are surely not the full story in string theory, they already encapsulate very rich behavior (including moduli stabilization, warping, supersymmetry breaking, etc.), and may exhibit much of the complexity of the full story. It is therefore heartening that simple conjectures like those of section 2 can essentially classify such large ensembles of vacua in simple terms. This increases our confidence that somewhere down the line, similar simple conjectures which capture basic aspects of the structure of the full space of string vacua may become available.

The most interesting physical feature to emerge is the existence of special loci in the moduli space which serve as attractors, in the sense that a large fraction of the flux vacua cluster around them. In the one-dimensional moduli space we examined here, the conifold plays this role, in a way that is immediately apparent from figure 3. The existence of such attractors in a more general setting could be important in helping to define a stringy notion of naturalness.

A useful generalization of this work would involve the study of models with many complex structure moduli. Finding concrete models which meet the requirements of [13] or its generalizations should also become possible, as we gain the ability to scan over large sets at once.[3] The ensemble where one focuses on IIB flux superpotentials could also be naturally enlarged to include generic Non-Kähler type II models [38], or classes of vacua where one includes an analysis of nontrivial D-brane worldvolume superpotentials (see [39] for a recent review). This could be important given that D-brane theories in flux backgrounds can give rise to very large numbers of vacua (see e.g. [40, 14]). And finally, it would be valuable to study other well-motivated toy ensembles which characterize the sets of supergravities emerging from others limits of M-theory, like the heterotic theory.

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[3] After this paper was completed, the interesting work [37] appeared where explicit examples along the lines of [13] are presented, including Kähler modulus stabilization.

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